four-membered transition state similar to that proposed[25] to account for bimolecular proton exchange in ketimines. It is also possible that some exchange might occur as the result of autoprotolysis of **1**. This seems unlikely to be important from the fact that the exchange is about equally fast in ethanol as in 50% ethanol.

### References and Notes

(1) Supported by the National Science Foundation, and by the Public Health Service, Research Grant GM-11073 from the Division of General Medical Sciences.
(2) R. P. Bell, "The Proton In Chemistry", 2nd ed, Cornell University Press, Ithaca, N.Y., 1973; E. Caldin and V. Gold, "Proton-Transfer Reactions", Chapman and Hall, London, 1975.
(3) M. Elgen, *Angew. Chem., Int. Ed. Engl.*, **3**, 1–19 (1964).
(4) M. Dreyfus, G. Dodin, O. Bensaude, and J. E. Dubois, *J. Am. Chem. Soc.*, **99**, 7027–7037 (1977); O. Bensaude, M. Dreyfus, G. Dodin, and J. E. Dubois, *ibid.*, **99**, 4438–4446 (1977); M. Dreyfus, O. Bensaude, G. Dodin, and J. E. Dubois, *ibid.*, **98**, 6338–6349 (1976).
(5) L. M. Jackman and F. A. Cotton, "Dynamic Nuclear Magnetic Resonance Spectroscopy", Academic Press, New York, N.Y., 1975, especially Chapter 15.
(6) E. Grunwald and E. K. Ralph, *Acc. Chem. Res.*, **4**, 107–113 (1971).
(7) A. J. Kresge, *Acc. Chem. Res.*, **8**, 354–360 (1975).
(8) J. R. Jones, *Prog. Phys. Org. Chem.*, **9**, 241–274 (1972).
(9) E. Grunwald, *Prog. Phys. Org. Chem.*, **3**, 317–358 (1965).
(10) F. M. Menger, H. K. Rhee, and J. U. Rhee, *J. Am. Chem. Soc.*, **98**, 792–796 (1976); J.-J. Delpuech and D. Nicole, *J. Chem. Soc., Perkin Trans. 2*, 570–576 (1977); K. Bouatouch, J. Chrisment, J.-J. Delpuech, and D. Nicole, *Org. Magn. Reson.*, **9**, 523–527 (1977).
(11) J.-J. Delpuech and D. Nicole, *J. Chem. Soc., Perkin Trans. 2*, 1025–1029 (1974).
(12) K. C. Chang, E. Grunwald, and L. R. Robinson, *J. Am. Chem. Soc.*, **99**, 3794–3796 (1977).
(13) F. M. Menger, T. D. Singh, and F. L. Bayer, *J. Am. Chem. Soc.*, **98**, 5013–5014 (1976).
(14) D. Gust and J. D. Roberts, *J. Am. Chem. Soc.*, **99**, 3637–3640 (1977).
(15) C. S. Irving and A. Lapidot, *J. Chem. Soc., Chem. Commun.*, 184–186 (1977).
(16) J. B. Lambert, G. Binsch, and J. D. Roberts, *Proc. Natl. Acad. Sci. U.S.A.*, **51**, 735–737 (1964); V. Markowski, T. B. Posner, P. Loftus, and J. D. Roberts, *ibid.*, **74**, 1308–1309 (1977); V. Markowskl, G. R. Sullivan, and J. D. Roberts, *J. Am. Chem. Soc.*, **99**, 714–718 (1977); G. C. Levy, C. E. Holloway, R. C. Rosanske, J. M. Hewitt, and C. H. Bradley, *Org. Magn. Reson.*, **8**, 643–647 (1976); G. E. Hawkes, E. W. Randall, and W. E. Hull, *J. Chem. Soc., Perkin Trans. 2*, 1268–1275 (1977).
(17) A. Fischer, D. A. R. Happer, and J. Vaughan, *J. Chem. Soc.*, 4060–4063 (1964).
(18) D. Gust, R. B. Moon, and J. D. Roberts, *Proc. Natl. Acad. Sci. U.S.A.*, **72**, 4696–4710 (1975).
(19) R. L. Lichter and J. D. Roberts, *J. Am. Chem. Soc.*, **94**, 4904–4906 (1972).
(20) J. M. Briggs, L. F. Farnell, and E. W. Randall, *J. Chem. Soc. D*, 680–681 (1971).
(21) E. V. Titov, L. M. Kapkan, and A. Yu. Chervinskii [*Theor. Eksp. Khim.*, **8**, 202–209 (1972)] have examined the 40-MHz $^1$H NMR spectrum of **1** in dimethyl sulfoxide at room temperature and report that the –NH– signal had a line width at half-peak height twice that of the –NH$_2$ signal. The difference in line width was attributed to the fact that the –NH$_2$ group has two exchangeable protons and thus the probability of proton transfer from this group is twice that from the –NH– group. However, the difference we observe between the two rates of proton transfer is much too large to be explained by statistical considerations used by Titov and co-workers.
(22) The present experimental data are not suitable for obtaining meaningful values of $\Delta H^{\ddagger}$ or $\Delta S^{\ddagger}$, even though the errors in $\Delta G^{\ddagger}$ are not large [F. A. L. Anet and R. Anet in "Determination of Organic Structures by Physical Methods", Vol. 3, F. C. Nachod and J. J. Zuckermann, Ed., Academic Press, New York, N.Y., 1971, pp 343–420; L. Lunazzi, A. Dondoni, G. Barbaro, and D. Macciantelli, *Tetrahedron Lett.*, 1079–1080 (1977)].
(23) The p$K_a$ of 2,2,2-trifluoroethanol is 12.8 [B. L. Dyatkin, E. P. Mochalina, and I. L. Knunants, *Tetrahedron*, **21**, 2991–2995 (1965)], while that of **1** is 5.20.[17] Therefore, we do not expect that **1** will be very extensively protonated. On the other hand, there is every reason to expect **1** to be considerably hydrogen bonded in this solvent. Because hydrogen bonding with the solvent is not per se expected to increase the rate of exchange by either of the mechanisms shown in eq 1 and 2, it seems likely that, in trifluoroethanol, the predominant exchange mechanism is the result of acid catalysis.
(24) F. Klages, K. Bott, and P. Hegebberg, *Angew. Chem., Int. Ed. Engl.*, **1**, 653–564 (1962).
(25) J. B. Lambert, W. L. Oliver, and J. D. Roberts, *J. Am. Chem. Soc.*, **87**, 5085–5090 (1965).

# Characterization of Anisotropic Motion in Fatty Acid Micelles by Analysis of Transverse Relaxation in an AX$_2$ Nuclear Spin System

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**Abstract:** An attempt is made to characterize the motion of a methylene group analogue in a fatty acid micelle using line shapes from a spin coupled $^{13}$C NMR spectrum. The analogue used is the fluorinated methylene of 6,6-difluorosodium myristate which under conditions of proton decoupling can be viewed as an AX$_2$ spin system with carbon-13 being the A spin. Expressions for line shapes in an AX$_2$ spin system in a non-extreme-narrowing limit are derived. It is observed that differences in widths of outer and inner lines of the $^{13}$C multiplet are simply related to the cross-correlation spectral density for the two carbon–fluorine dipolar interactions. The observed sign and magnitude of the cross-correlation density is compared to predictions based on models having isotropic, anisotropic, or restricted internal motions.

### Introduction

Motional processes can in principle be characterized at a molecular level through the measurement of longitudinal and transverse relaxation phenomena in magnetic resonance. These measurements sample spectral density functions associated with temporal fluctuations in the interactions responsible for relaxation. The discrete frequencies which are sampled depend in part on the magnetic field strength chosen for the experiment.

In simple cases, such as two isolated spins at fixed internuclear distance undergoing isotropic reorientation by a Brownian process, a single sampling such as a $T_1$ measurement is often sufficient to characterize the time scale of motion. In more complex cases, for example, reorientation of a methylene of a hydrocarbon chain in the semiordered array of a micelle or biological membrane, analysis is less straightforward. In these instances the assumption of isolated spin groupings may be invalid, and an adequate rotational model could require full consideration of both restricted and anisotropic molecular motion. Complete characterization of time parameters for such

motional models requires the sampling of spectral density functions at several frequencies and requires the proper treatment of multispin correlated motions.

The proper treatment of correlated motions of three or more interacting spins is particularly important as both autocorrelation and cross-correlation spectral density functions must be included. For a pair of dipolar interaction vectors in a carbon-13 containing methylene group the cross-correlation function depends upon the HCH angle and in essence describes the degree to which the motion of one internuclear vector correlates with the other during motional reorientations. The motion of two spatially coupled vectors requires the time description of at least three positional angles, whereas the orientation of a single vector needed to specify the autocorrelation function may be characterized by the time dependence of only two positional angles. Thus, if cross-correlation spectral densities can be independently determined, it might be anticipated that they would provide a more stringent test of an assumed motional model than autocorrelation densities alone.

A carbon-13 containing methylene group of a hydrocarbon chain, assuming that it is sufficiently isolated from adjacent groups and chains, will approach an $AX_2$ system. A complete treatment of spin–lattice or longitudinal relaxation for an $AX_2$ system has recently appeared[2] and has been employed in the analysis of small molecule motion in both solution[3] and liquid-crystal environments.[4] Expressions which include cross-correlation for spin–spin or transverse relaxation in the extreme narrowing limit for $AX_2$ systems have also appeared.[5,6] Measurement of transverse relaxation for small molecules is difficult, however, so few applications of the theory have been attempted.

Presented here is a refinement of the theory for transverse relaxation in $AX_2$ systems for application to longer time scale motions $(1/\omega_0 \ll \tau_c \ll \hbar/\mathcal{H}_D)$ as well as an application to the analysis of motional properties of a methylene group analogue in the semiordered array of a fatty acid micelle. Measurement of transverse relaxation in these aggregated systems is in fact straightforward since the parameters can be extracted directly from line widths of a single spectrum. The measurements are doubly valuable because they are likely to display primary sensitivity to slow motions which are often minor contributions to longitudinal relaxation.

In the interest of spectral simplicity and resolution we will actually study the $^{13}$C spectrum of a $CF_2$ group near the middle of a fatty acid chain. It is assumed that under conditions of proton decoupling the group can be treated as a simple $AX_2$ system and that the motion of the $CF_2$ group is representative of the motional features of a methylene group at a corresponding position in a nonfluorinated fatty acid chain.

## Theory

The energy level diagram showing eigenfunctions and numbering of states for an $AX_2$ system is given in Figure 1. We will be interested primarily in the "A" or carbon-13 portion of the spectrum consisting of a 1:2:1 triplet with the high to low field lines associated with transitions 2–1, 6–3 plus 5–4, and 8–7, respectively.

Quantitative descriptions of magnetic relaxation in a system such as this are most frequently approached using Redfield–Bloch theory.[7,2] Following this approach the time evolution of a parameter of interest such as the transverse magnetization, $M_x$, in a frame rotating at the resonance frequency of a particular line can be expressed as follows:

$$\frac{d}{dt} \langle\alpha|M_x(t)|\alpha'\rangle = \sum_{\alpha''\alpha'''} R_{\alpha\alpha'\alpha''\alpha'''}\langle\alpha''|M_x(t)|\alpha'''\rangle \quad (1)$$

Here, $\alpha$, $\alpha'$, $\alpha''$, $\alpha'''$ denote the various states depicted in Figure 1, and the $R_{\alpha\alpha'\alpha''\alpha'''}$ are elements of the complete relaxation
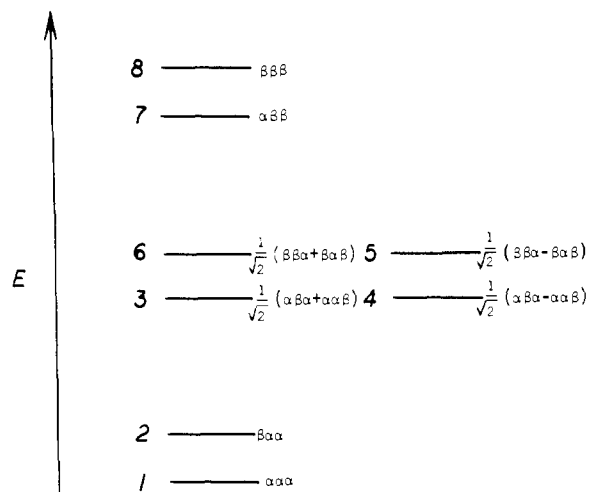


Figure 1. Energy level diagram for an $AX_2$ system.

matrix. Assuming that the three lines of the triplet are well resolved, eq 1 reduces to the following expression for the outer line:

$$\frac{d}{dt}\langle1|M_x(t)|2\rangle = R_{1212}\langle1|M_x(t)|2\rangle \quad (2)$$

and to the following expressions for the symmetric and antisymmetric combinations of the component magnetizations of the central line.

$$\frac{dM_+(t)}{dt} = \frac{d}{dt}(\langle3|M_x(t)|6\rangle + \langle4|M_x(t)|5\rangle)$$

$$= \frac{1}{2}(R_{3636} + 2R_{3645} + R_{4545})M_+(t)$$

$$+ \frac{1}{2}(R_{3636} - R_{4545})M_-(t) \quad (3)$$

$$\frac{dM_-(t)}{dt} = \frac{d}{dt}(\langle3|M_x(t)|6\rangle - \langle4|M_x(t)|5\rangle)$$

$$= \frac{1}{2}(R_{3636} - R_{4545})M_+(t)$$

$$+ \frac{1}{2}(R_{3636} - 2R_{3645} + R_{4545})M_-(t) \quad (4)$$

Only the symmetric combination can be observed in the experiments presented here, but the antisymmetric combination can be observed in spin tickling experiments or experiments conducted in anisotropic environments such as those used recently by Courtieu et al.[4]

Evaluation of the five Redfield relaxation terms appearing in eq 2–4 depends on an assumed relaxation mechanism. For dipolar interactions contributions from various pairs of interacting spins, $ij$, $kl$, can be expressed as follows:

$$R_{\alpha\alpha'\alpha''\alpha'''}^{ijkl}$$

$$= \sum_{m,n}\left\{2J_{ijkl}^{mn}(\omega_{\alpha\alpha''})\langle\alpha|T_2^m(ij)|\alpha''\rangle\langle\alpha'''|T_2^n(kl)|\alpha'\rangle\right.$$

$$- \delta_{\alpha\alpha''}\sum_{\beta}J_{ijkl}^{mn}(\omega_{\alpha'''\beta})\langle\alpha'''|T_2^m(ij)|\beta\rangle\langle\beta|T_2^n(kl)|\alpha'\rangle$$

$$\left. - \delta_{\alpha'\alpha'''}\sum_{\beta}J_{ijkl}^{mn}(\omega_{\alpha''\beta})\langle\alpha|T_2^m(ij)|\beta\rangle\langle\beta|T_2^n(kl)|\alpha''\rangle\right\} \quad (5)$$

The time-dependent dipolar interaction $\mathcal{H}_D(t)$ has been expanded in spherical tensor notation.[2] The $T_2^{n(m)}$ are the components of the second-order spin tensor with $n$ or $m$ running from $-2$ to 2. The $J_{ijkl}^{mn}(\omega_{\alpha\alpha''})$ are the second-order spherical harmonic spectral densities at frequency $\omega_{\alpha\alpha} = (E_{\alpha''} - E_\alpha)/\hbar$.
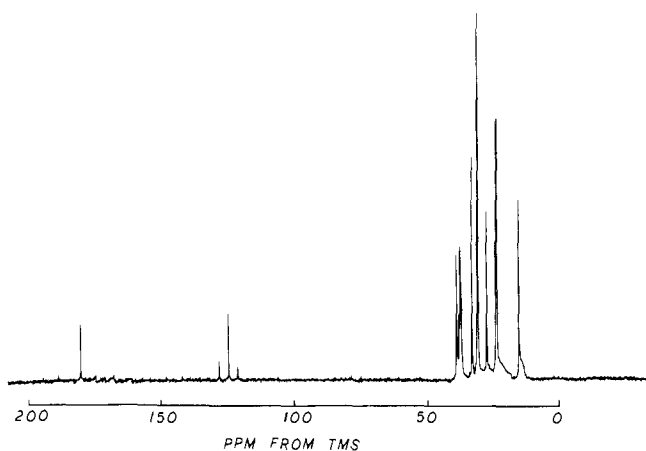
**Figure 2.** 67.9-MHz spectrum of an aqueous micellar dispersion of 6,6-difluorosodium myristate at 44 °C.

The elements in eq 2–4 have been evaluated assuming an isotropic distribution of AX vectors with respect to the external field to give the following results:

$$R_{1212} = \frac{-4}{3} J_{AX_1AX_1}(0) - \frac{1}{3} J_{AX_1AX_1}(\omega_A - \omega_X)$$

$$- J_{AX_1AX_1}(\omega_A) - J_{AX_1AX_1}(\omega_X) - 2J_{AX_1AX_1}(\omega_A + \omega_X)$$

$$- \frac{4}{3} J_{X_1AAX_2}(0) - J_{X_1AAX_2}(\omega_A) - J_{X_1X_2X_1X_2}(\omega_X)$$

$$- 2J_{X_1X_2X_1X_2}(2\omega_X) \quad (6)$$

$$R_{3636} = -\frac{2}{3} J_{AX_1AX_1}(0) - \frac{1}{3} J_{AX_1AX_1}(\omega_A - \omega_X)$$

$$- J_{AX_1AX_1}(\omega_A) - J_{AX_1AX_1}(\omega_X) - 2J_{AX_1AX_1}(\omega_A + \omega_X)$$

$$+ \frac{2}{3} J_{X_1AAX_2}(0) - \frac{1}{3} J_{X_1AAX_2}(\omega_A - \omega_H) + J_{X_1AAX_2}(\omega_A)$$

$$- J_{X_1AAX_2}(\omega_X) - 2J_{X_1AAX_2}(\omega_A + \omega_X) - 2J_{X_1X_2X_1X_2}(\omega_X)$$

$$(7)$$

$$R_{4545} = -\frac{2}{3} J_{AX_1AX_1}(0) - \frac{1}{3} J_{AX_1AX_1}(\omega_A - \omega_X)$$

$$- J_{AX_1AX_1}(\omega_A) - J_{AX_1AX_1}(\omega_X) - 2J_{AX_1AX_1}(\omega_A + \omega_X)$$

$$+ \frac{2}{3} J_{X_1AAX_2}(0) + \frac{1}{3} J_{X_1AAX_2}(\omega_A - \omega_X) + J_{X_1AAX_2}(\omega_A)$$

$$+ J_{X_1AAX_2}(\omega_X) + 2J_{X_1AAX_2}(\omega_A + \omega_X) \quad (8)$$

$$R_{3645} = R_{4536} = -\frac{2}{3} J_{AX_1AX_1}(0) + \frac{2}{3} J_{X_1AAX_2}(0) \quad (9)$$

All $J^{mn}$ have been expressed in terms of $J^{00}$ dropping the 00 superscript.

For the outer lines of the triplet, the magnetization decays with a single time constant $R_{1212}$ which can be associated with a single $1/T_2$. The lines should be Lorentzian with full width at half-height equal to $-\pi^{-1}R_{1212}$. Relaxation of magnetization for the central line will usually be more complex. The symmetric and antisymmetric combinations $M_+$ and $M_-$ are coupled by the term $(R_{3636} - R_{4545})$ giving a non-Lorentzian peak shape for $M_+$ which cannot be characterized by a single $1/T_2$. It is observed, however, that $(R_{3636} - R_{4545})$ contains no zero frequency spectral density contributions, so that if spectral densities at zero frequency are much greater than at other frequencies (i.e., $\omega_A - \omega_X$, $\omega_A$, $\omega_X$, $\omega_A + \omega_X$, and $2\omega_X$), both lines will exhibit Lorentzian behavior. In this slow motion

limit one obtains for the outer line:

$$\frac{d}{dt} \langle 1 | M_x(t) | 2 \rangle$$

$$= \left( -\frac{4}{3} J_{AX_1AX_1}(0) - \frac{4}{3} J_{X_1AAX_2}(0) \right) \langle 1 | M_x(t) | 2 \rangle \quad (10)$$

and for the central line symmetric combination:

$$\frac{d}{dt} M_+(t) = \left( -\frac{4}{3} J_{AX_1AX_1}(0) + \frac{4}{3} J_{X_1AAX_2}(0) \right) M_+(t)$$

$$(11)$$

The central line antisymmetric combination contains no zero frequency contributions. It is clear that, although both lines in this limit should be single Lorentzian, widths could differ greatly. The difference in width between central and outer components is simply $(-8/\pi3)J_{X_1AAX_2}(0)$ and gives a measure of the cross-correlation spectral density at zero frequency. This observation is significant for the study of large molecules or aggregated systems, such as membranes and micelles, since one often operates in a region where $J(0) \gg J(\omega)$ as evidenced by the fact that $T_2 \ll T_1$.

## Experimental Section

6,6-Difluorosodium myristate in micellar dispersion proves to be a suitable subject for the study of a pseudo-AX$_2$ group in a hydrocarbon chain environment. The *gem*-difluoro fatty acid has been synthesized previously using $MoF_6$ and experimental details of that synthesis have been published.[8] Our synthesis followed the published procedure except that the keto acid precursor was synthesized using the method of Cason and Prout[9] with octyl bromide and the monomethyl ester of adipic acid (purchased from Aldrich) as starting materials. Isolation of the difluoro acid from the fluorination reaction mixture was accomplished by saponification of the fatty ester followed by crystallization of a crude mixture of keto and fluoro acids from acetonitrile. This in turn was followed by chromatographic separation on silicic acid using petroleum ether–ether–acetic acid (80:20:1).

5,5,7,7-Tetradeuterio-6,6 difluoromyristic acid was also synthesized. It was deuterated (>80%) at the positions vicinal to the fluorines by refluxing the methyl keto ester precursor for 4 h in 2 M NaOD, and reesterifying the deuterated keto acid by reflux for 5 h in a 20-fold excess of methanol-$d_1$ (99% from Aldrich) containing a slight molar excess of boron trifluoride etherate. Loss of some deuteriums when deuterated methanol was not used in the esterification indicated that it would be possible to deuterate directly with the BF$_3$ reagent eliminating the NaOD step.

Micellar samples were prepared by dissolving the *gem*-difluoro acids in D$_2$O containing 1 molar equiv of NaOH and being ~2 mM in EDTA. Final concentrations were 1.7 *m* except where noted. At this concentration the sample undergoes a transition from a gel to a micellar phase at approximately 35 °C. Samples were transferred to 10-mm NMR tubes and examined at temperatures above this point.

Carbon-13 spectra of the samples were obtained at two field strengths. A Bruker HX270 spectrometer operating at 63 kG provided the higher field. Noise decoupling of protons during acquisition was employed in the case of the nondeuterated acid. The field was locked to the D$_2$O deuterium resonance and spectra were acquired with an 8-s repetition rate. For the study of line shapes of the CF$_2$ triplet the frequency domain was reduced to 2000 Hz with an 8K data set to maximize digital resolution. Use of quadrature detection with a 500-Hz low pass active filter minimized spectral foldover. Approximately 1500 transients were accumulated.

Low-field spectra were obtained using a Varian XL100. Conditions for acquisition were similar except that continuous single frequency decoupling was employed. Experiments at the higher field indicated that the precise decoupling mode did not influence line widths within the precision required here. Temperatures could be reproduced with a precision of ±3 °C on the two spectrometers.

## Results

A 67.9-MHz $^{13}$C spectrum of 6,6-difluorosodium myristate in micellar dispersion is presented in Figure 2. The multiplet
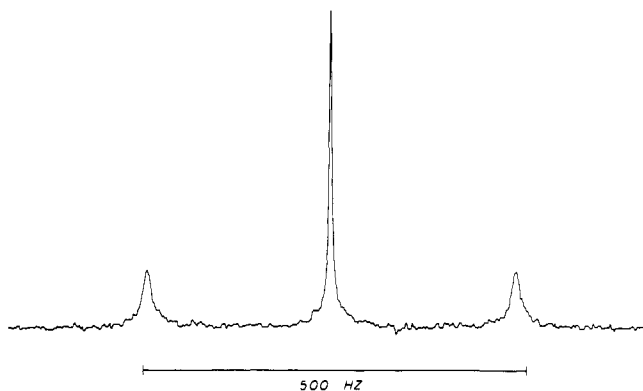
Figure 3. $CF_2$ triplet of 6,6-difluorosodium myristate at 67.9 MHz and 44 °C under conditions of proton noise decoupling.



Figure 4. $CF_2$ triplet of 5,5,7,7-tetradeuterio-6,6 difluorosodium myristate at 67.9 MHz and 44 °C without proton decoupling.

near 125 ppm coming from the $CF_2$ group at midchain is well separated from the spectral region of the other methylenes. Under conditions of proton decoupling the $^{13}CF_2$ resonance lines appear as a triplet with a 241-Hz C–F coupling. We will assume that this triplet behaves as the A part of an $AX_2$ system.

A spectrum collected at 67.9 MHz and 44 °C using a narrow spectral window to focus on the $CF_2$ group is shown in Figure 3. It is clear that, although the three peaks show the expected 1:2:1 area ratio, there is a dramatic variation in line widths. The widths measured at half-height are 16 and 5 Hz, respectively, for the outer and central components. Magnet inhomogeneity may contribute slightly to the non-Lorentzian baseline around the central peak, but, since widths of homogeneity limited peaks are normally much less than 1 Hz, measurements at half-height can be taken as indicative of the properties of the sample. The widths of both outer and inner lines are very temperature dependent, changing to 8 and 6 Hz at 70 °C. A spectrum measured at 25 MHz and 44 °C shows a phenomenon similar to the 44 °C high-field spectrum with a small field dependence to the line widths. The inner and outer lines measure 24 and 6 Hz, respectively.

Spin–lattice relaxation of the spectrum at 25 MHz was also examined using a 2.0 $m$ sample at 47 °C. Although no single exponential time constant can describe relaxation for all lines, relaxation of the sum of intensities can be characterized by a time constant of 1.2 s. It is clear that $T_2 \ll T_1$.

It is difficult to assess the possible influence of noise decoupling of protons during acquisition; however, the question can be approached experimentally by replacing vicinal protons with deuteriums. This does not eliminate all observable proton coupling to the carbon of the $CF_2$ group since two- and three-bond coupling constants are of nearly equal magnitude. With only three-bond coupling observable, however, even proton coupled lines in the $CF_2$ triplet are narrow enough to show a difference between outer and inner components (Figure 4). Replacement of vicinal protons with deuteriums also reduces non-C–F dipolar interactions to a point where they should not be of concern. The qualitative similarities between deuterated and nondeuterated samples lend confidence to our treatment of the $CF_2$ group as an $AX_2$ spin system.

## Discussion

The difference in transverse relaxation for outer and central lines of the triplet presented in the previous section can be interpreted in terms of contributions from various parts of the spectral density functions. This is most easily done if we are in the slow motion limit where $J(0)$ terms dominate. That we are near this limit in the 44 °C data is suggested by the fact that transverse relaxation is much faster than longitudinal
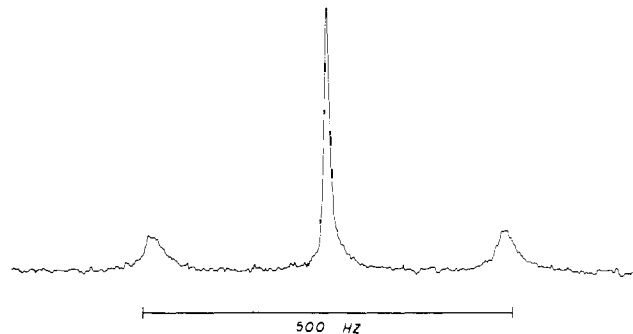
relaxation and also by the small frequency dependence of line widths in going from 67.9 to 25 MHz.

The frequency dependence observed near the slow-motion limit can in principle be used to estimate a correlation time for molecular reorientation, $\tau_c$, by including the next most important spectral density function contributions, $J(\omega_A)$, in eq 10 and 11. For the outer line the expression becomes

$$\frac{d}{dt} \langle 1|M_x(t)|2\rangle = \left\{ -\frac{4}{3} J_{AX_1AX_1}(0) - \frac{4}{3} J_{X_1AAX_2}(0) \right.$$
$$\left. - J_{AX_1AX_1}(\omega_A) - J_{X_1AAX_2}(\omega_A) \right\} \langle 1|M_x(t)|2\rangle \quad (12)$$

If we assume the time decay of all the correlation functions to be exponential and to have the same time constant, we estimate $\tau_c$ to be $5 \times 10^{-9}$ s. Contributions of $J(\omega)$ at higher frequency neglected in this analysis make contributions of less than 5% in the high-field case and less than 20% in the lower field case.

The correlation time obtained, if associated with rotation of a rigid spherical micelle according to the Stokes formula, would be that of a micelle with radius 17 Å. This is reasonable in light of figures for maximum minor axis radius of a micelle formed from a 14-carbon chain estimated on the basis of a formula given by Tanford.[10] Such a small micelle, however, should not give line widths as large as those observed here if rapid internal chain motions exist. A $\tau_c$ on the order of $10^{-7}$ would be required. It is possible that the correlation time calculated on the basis of eq 12 is an underestimate because of the large temperature coefficient of observed line widths and the difficulty in reproducing thermal conditions on two different spectrometers. It is also probable that the micelles are nonspherical in shape at the fatty acid concentrations studied here. The estimate of a relatively long correlation time can, however, be used to support application of eq 10 and 11 in further analyses.

Given that $\tau_c$ is sufficiently long, at least at the high field strength, to make eq 10 and 11 valid we should be able to determine both $J_{CFCF}$ and $J_{FCCF}$. Although there is no unique way of going from these density functions to a model for molecular motion, we can, nevertheless, use measurements of these functions to dismiss or retain reasonable proposals for motion. For example, expressions for both autocorrelation and cross-correlation spectral density functions have been derived for rigid rotating ellipsoids.[2] In the spherical limit one finds that $J_{FCCF} = -0.34 J_{CFCF}$. Since the functions are of opposite sign, one would predict that the outer lines of the triplet would be narrower than the inner lines. This is clearly not the case for our $CF_2$ group and an isotropic rigid rotor can be dismissed as an appropriate model. Our observed relative line widths can be reproduced if $J_{FCCF} = 0.52 J_{CFCF}$. Outer lines would then be slightly more than a factor of 3 wider than central lines. One case in which this occurs is for a rotationally diffusing ellipsoid, long axis perpendicular to the CF bond, with an axial ratio of

approximately 10:1. A model in which the micelle itself is the ellipsoid can be dismissed as physically unreasonable as the chain axis is certainly parallel to the minor axis in any nonspherical micelle.

Models which include restricted internal motions as well as overall anisotropic or isotropic motions are more appropriate for description of hydrocarbon chain motion. Such models have been proposed by Petersen and Chan[11] and by Gent and Prestegard.[12] In the latter model rapid internal bond isomerizations characterized by correlation time $\tau_D$ are allowed to rapidly reorient CH or CF vectors through all angles about the average chain axis, but only through angles within $\pm\Delta$ of a perpendicular to the chain axis, where $\Delta$ is approximately 60°. Superposition of slower isotropic chain axis reorientation characterized by correlation time $\tau_c$ allows sampling of all vector orientations in times short compared to $\hbar/\mathcal{H}_D$. Autocorrelation and cross-correlation spectral density functions for this model are given as follows:

$$J_{CFCF} = \frac{3}{10} \frac{\gamma_c^2 \gamma_F^2 \hbar^2}{r_{CF}^6} \left\{ \left(1 - \frac{1}{4}\cos^4\Delta\right) \frac{\frac{1}{\tau_{D'}}}{\left(\frac{1}{\tau_{D'}}\right)^2 + \omega^2} \right.$$

$$\left. + \frac{1}{4}\cos^4\Delta \frac{\frac{1}{\tau_c}}{\left(\frac{1}{\tau_c}\right)^2 + \omega^2} \right\} \quad (13)$$

$$J_{FCCF} = \frac{3}{10} \frac{\gamma_c^2 \gamma_F^2 \hbar^2}{r_{CF}^6} \left\{ \left(1 - 6\cos^2\frac{\epsilon}{2} + 6\cos^4\frac{\epsilon}{2} - \frac{\cos^4}{4}\Delta\right) \right.$$

$$\left. \times \frac{\frac{1}{\tau_{D'}}}{\left(\frac{1}{\tau_{D'}}\right)^2 + \omega^2} + \frac{\cos^4\Delta}{4} \frac{\frac{1}{\tau_c}}{\left(\frac{1}{\tau_c}\right)^2 + \omega^2} \right\} \quad (14)$$

where $\epsilon$ is the FCF angle, $\gamma_c$ and $\gamma_F$ are the magnetogyric ratios of carbon and fluorine, respectively, $(\tau_{D'})^{-1} = (\tau_D)^{-1} + (\tau_c)^{-1}$, and $r_{CF}$ is the internuclear distance. The expressions are derived under the additional assumption that the two CF vectors make equal and opposite departures from the chain axis perpendicular, but this assumption only affects coefficients of terms containing an $\epsilon$ dependence.

Under conditions where $\tau_D \ll \tau_c$ and $\Delta \neq 90$ both expressions for $J(0)$ converge to the following:

$$J_{CFCF}(0) = J_{FCCF}(0) = \frac{3}{40} \frac{\gamma_c^2 \gamma_F^2 \hbar^2}{r_{CF}^6} \cos^4\Delta\tau_c \quad (15)$$

Thus, this motionally restricted model predicts that the outer lines of an $AX_2$ triplet should be much broader than the central line with the central line becoming infinitely narrow under the assumptions used here. Central lines are, of course, never infinitely narrow and can, in fact, broaden to the point of being

one-third the width of an outer line via mechanisms perfectly consistent with the restricted internal motion model. First, spectral density contributions at nonzero frequency will always add to the width of the central line. Second, spin–spin relaxational contributions other than C–F dipolar interactions may be present. And third, $\tau_c$ and $\tau_D$ may not satisfy the assumption that $\tau_D \ll \tau_c$. Thus, the restricted motion model produces a reasonable qualitative description of $CF_2$ reorientation in the semiordered hydrocarbon array of a fatty acid micelle.

A quantitative evaluation of $\tau_c$ and $\tau_D$ from multiplet line widths can be pursued on the basis of eq 10, 11, 13, and 14, if one assumes that central line width arises because $\tau_D$ is not sufficiently small compared to $\tau_c$. It is necessary to choose a value for the restriction angle $\Delta$. If 60°, as suggested for a chain undergoing $\beta$-coupled isomerizations, is used, $\tau_c$ must be 6 × $10^{-7}$ s and $\tau_D$ 6 × $10^{-9}$ s. Because of other possible contributions to the central line width, $\tau_D$ should be regarded as an upper limit; 6 × $10^{-9}$ s is, in fact, long for the observed spin–lattice relaxation time.

Complete description of chain motion can obviously be approached only by accurately determining both transverse and longitudinal relaxation times. It is, however, significant that in many cases a qualitative statement about the existence of anisotropic or restricted motion can be made on the basis of a single spectral observation. In this case the $CF_2$ group clearly exhibits restricted or highly anisotropic internal motion. Large molecule systems or systems with high degrees of aggregation where transverse relaxation is reflected in line widths are precisely those where determination of longitudinal relaxation of methylene carbons is difficult because of the long time periods required for sequential spectral accumulations. We, therefore, expect transverse relaxation analysis of $AX_2$ systems to open many applications in the determination of the structure of macromolecules.

**References and Notes**

(1) (a) Yale University; (b) University of Utah.
(2) L. G. Werbelow and D. M. Grant, *Adv. Magn. Reson.* **9**, 189–299 (1977).
(3) C. L. Mayne, D. M. Grant, and D. W. Alderman, *J. Chem. Phys.*, **65**, 1684 (1976).
(4) J. Courtieu, P. E. Fagerness, and D. M. Grant, *J. Chem. Phys.*, **66**, 2669 (1977).
(5) W. M. M. J. Bovee, *Mol. Phys.*, **29**, 1673 (1975).
(6) R. R. Vold and R. L. Vold, *J. Chem. Phys.*, **64**, 320 (1976).
(7) C. P. Slichter, "Principles of Magnetic Resonance", Harper and Row, New York, N.Y., 1963, Chapter 5.
(8) M. P. N. Gent, I. M. Armitage, and J. H. Prestegard, *J. Am. Chem. Soc.*, **98**, 3749 (1976).
(9) J. Cason and F. S. Prout in "Organic Syntheses", Collect. Vol. 3, E. C. Horning, Ed., Wiley, New York, N.Y., 1955, pp 601–604.
(10) C. Tanford, "The Hydrophobic Effect", Wiley, New York, N.Y., 1973, p 75.
(11) N. O. Petersen and S. I. Chan, *Biochemistry*, **16**, 2657 (1977).
(12) M. P. N. Gent and J. H. Prestegard, *J. Magn. Reson.*, **25**, 243 (1977).